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From Production Functions to Frontiers: A Critical Review of Productivity and Efficiency Measurement in Higher Education

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Abstract

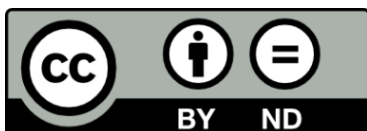
Improving productivity in education has become a central policy concern, particularly in public education systems facing fiscal constraints and rising accountability pressures. Education Production Function (EPF) studies provide the dominant analytical framework for examining how educational inputs are transformed into learning outcomes. However, despite decades of research, there is no clear consensus on how to measure productivity and efficiency in education, largely due to methodological fragmentation across studies. The paper provides a critical and integrative review of the education production function literature, synthesizing theoretical foundations, empirical model specifications, and estimation strategies employed in productivity measurement, including contemporaneous, value-added, and growth EPF models. The review evaluates regression-based approaches and frontier methods such as Stochastic Frontier Analysis and Data Envelopment Analysis, with particular attention to methodological assumptions, data requirements, and sources of bias. The study contributes by linking production function models with efficiency analysis and identifying major sources of empirical inconsistency in EPF studies. It emphasises the importance of multi-output, joint production, and hybrid approaches for improving productivity measurement. The review shows that conflicting findings mainly arise from methodological problems such as endogeneity, omitted variable bias, misspecification, restrictive functional forms, and poor treatment of multiple outputs. Although value-added and frontier models improve on simple regressions, their reliability depends on data quality and assumptions. The study highlights the need for integrated, dynamic, and multidimensional approaches, especially in higher education and developing-country contexts, to support evidence-based education policies.

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INTRODUCTION

Improving productivity in education has become a major policy concern, particularly in publicly funded systems where limited resources must be used efficiently to meet increasing demand and accountability pressures (Agasisti & Johnes, 2015; Hanushek, 2003). In economic terms, productivity refers to the efficiency with which inputs are transformed into outputs, making productivity measurement essential for evaluating performance, identifying inefficiencies, and guiding resource allocation (Organisation for Economic Co-Operation and Development [OECD], 2001). In education, these issues are especially important because of the sector's public financing and its broader social and economic contributions (Becker, 1962; Sullivan et al., 2012).

The application of microeconomic production theory to education led to the development of the education production function (EPF) framework, which views education as a process where inputs such as student characteristics, institutional resources, and teaching quality are transformed into educational outcomes (Hanushek, 1979, 1986). Since the publication of the Coleman Report (Coleman et al., 1966), EPF models have become a dominant analytical framework in the economics of education for examining student achievement and institutional performance.

Despite their extensive use, there is still no consensus on how productivity and efficiency in education should be measured. Empirical findings remain highly inconsistent across studies due to methodological fragmentation, including differences in model specification, data structure, treatment of endogeneity, and handling of dynamic learning processes and multiple outputs (Figlio, 1999; Hanushek, 1979, 1986; Todd & Wolpin, 2003). Consequently, studies often reach conflicting conclusions about the effectiveness of educational inputs, limiting their value for policy formulation.

Researchers have introduced value-added models, panel data techniques, and frontier-based efficiency approaches such as stochastic frontier analysis (SFA) and data envelopment analysis (DEA) to address these limitations (Aigner et al., 1977; Charnes et al., 1978; De Witte & López-Torres, 2017; Hanushek & Rivkin, 2010). However, these methods are frequently applied independently, without integration into a unified analytical framework. In addition, many studies continue to rely on single-output models, despite education producing multiple outcomes, including cognitive skills, non-cognitive abilities, and labour-market outcomes (Heckman & Kautz, 2012; Kounetas et al., 2023; OECD, 2018). As a result, the literature lacks a comprehensive framework that links production function models to efficiency analysis while accounting for

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joint production and dynamic learning processes.

Against this background, the present study provides a critical and integrative review of EPF studies with particular emphasis on productivity measurement. The review aims to: (i) synthesise the theoretical foundations and empirical specifications of EPF models; (ii) evaluate methodological approaches, including regression-based and frontier efficiency techniques; (iii) identify major sources of empirical inconsistency and ambiguity; and (iv) examine emerging approaches such as multi-output, joint production, and hybrid models for improving productivity analysis, especially in higher education and developing-country contexts.

The remainder of the paper is organised as follows. Section 2 presents the conceptual framework. Section 3 reviews alternative EPF models, while Section 4 examines methodological approaches to productivity measurement, including regression and frontier techniques. Section 5 discusses joint production and multiple-output frameworks. Section 6 synthesises the empirical evidence, and Section 7 considers policy implications and future research directions. Finally, Section 8 concludes the review.

MATERIALS AND METHODS

This study adopts a structured narrative review approach to

synthesise the literature on EPF, productivity, and efficiency analysis in education, with a particular emphasis on higher education contexts. While not a formal meta-analysis, the review follows a transparent and systematic procedure for identifying, selecting, and analysing relevant studies.

The literature was collected from major academic databases, including Scopus, Web of Science, Google Scholar, and EconLit, which are widely used in economics and education research. The search was conducted using combinations of the following keywords: “education production function,” “education productivity,” “efficiency in education,” “stochastic frontier analysis in education,” “data envelopment analysis in education,” “value-added models in education,” and “higher education efficiency.” The search focused primarily on peer-reviewed journal articles, supplemented by influential books, policy reports (e.g., OECD), and seminal working papers, where relevant.

Studies were included in the review based on the following criteria; (1) Direct relevance to education production functions, productivity, or efficiency measurement, (2) Application of quantitative or formal modelling approaches (e.g., regression, SFA, DEA, dynamic models), (3) Empirical or methodological contribution to education productivity analysis, and (4) Publication in peer-reviewed journals or recognized



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academic sources. Studies were excluded if they (1) focused purely on descriptive education outcomes without a production or efficiency framework, (2) lacked methodological clarity, and (3) were not accessible in full text.

The article selection process followed the PRISMA 2020 guidelines (Page et al., 2021) to ensure transparency and replicability.

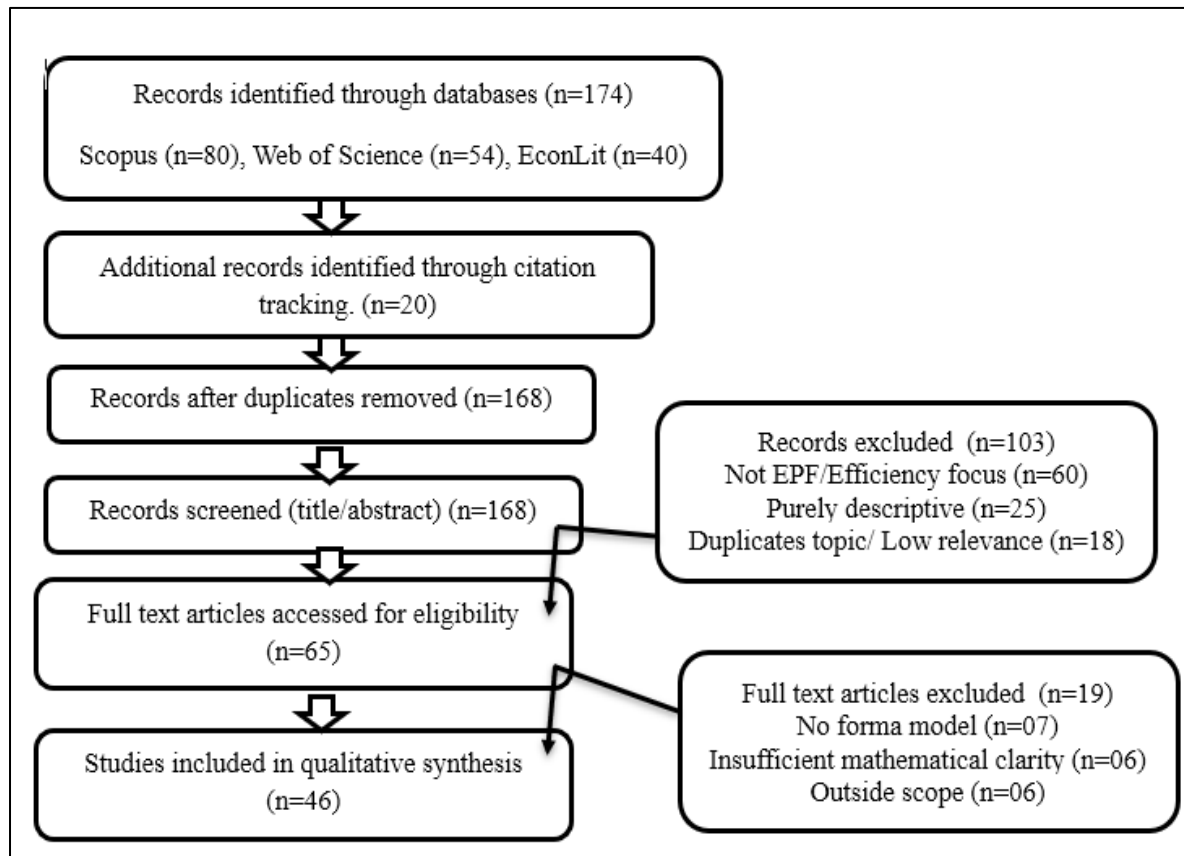


Figure 1: PRISMA 2020 flow diagram of the study selection process

The review involved four stages: identification, screening, eligibility assessment, and inclusion. A total of 194 records were identified through database searches and citation tracking, including 174 records from Scopus (80), Web of Science (54), and EconLit (40), along with 20 additional studies identified through backward and forward citation tracking. After removing duplicates, 168 records

remained for title and abstract screening, during which 103 studies were excluded due to limited relevance to education production functions, productivity, or efficiency analysis. The remaining 65 full-text articles were assessed for eligibility, and 19 were excluded because they lacked formal production or efficiency models, methodological rigor, or relevance to the review objectives. Ultimately, 46

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studies were included in the qualitative synthesis, representing major empirical and methodological contributions to the EPF literature.

Conceptualising Productivity in Education

In economics, productivity refers to how efficiently inputs are transformed into outputs. The OECD (2001) defines productivity as the ratio of output volume to input volume, highlighting its importance in measuring technical change, efficiency improvement, benchmarking, and cost reduction. Although initially developed for industrial sectors, the concept has increasingly been applied to service sectors such as education, where outputs are less tangible and production processes are more complex.

In education, productivity involves transforming inputs such as student characteristics, family background, peer influences, institutional resources, and teaching inputs into educational outcomes (Belley et al., 2014; Hanushek, 1979, 1986; Sullivan et al., 2012). Rolle (2004) describes education productivity as the efficient production of educational outcomes within existing resource constraints. Given growing demands for quality and equity under limited budgets, productivity has become a major concern in publicly funded education systems (OECD, 2001).

Educational productivity is closely linked with efficiency and accountability. Technical efficiency refers to producing maximum educational outputs from given inputs, while allocative efficiency concerns the optimal use of resources relative to their costs (Aigner et al., 1977; Hanushek, 1986). These concepts have gained increasing attention as governments seek measurable educational and social returns from public investment in education (Hanushek & Woessmann, 2015).

Productivity analysis has become increasingly important in higher education due to rising enrolments, growing unit costs, and concerns regarding graduate employability (Agasisti & Johnes, 2015; Archibald & Feldman, 2018; Deming, 2017; Sullivan et al., 2012). Sullivan et al. (2012) argue that higher education institutions face a persistent trade-off between affordability and quality unless productivity improves through innovations in instructional technology and delivery methods. However, productivity growth in education has remained slower than in many other sectors because of limited technological integration and rigid institutional structures (Archibald & Feldman, 2018).

A major challenge in education productivity analysis is defining educational outputs. Traditional productivity models often rely on measurable outcomes such as test scores and examination results, but

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these indicators capture only a limited dimension of education (Hanushek, 1979; Summers & Wolfe, 1977). Education also generates broader outcomes, including non-cognitive skills, attitudes, social behaviours, and labour-market success (Cunha et al., 2010; Deming, 2017; Heckman & Kautz, 2012). Consequently, recent literature increasingly emphasises multidimensional output measures, particularly in higher education and skill development contexts.

A major challenge in educational productivity analysis is incorporating the cumulative and dynamic nature of learning processes. Educational outcomes are influenced not only by current inputs but also by past investments made by families, schools, and communities, as well as unobserved individual endowments (Ben-Porath, 1967; Cunha et al., 2010; Todd & Wolpin, 2003). Therefore, learning should be viewed as a cumulative process, requiring dynamic rather than static productivity models (Todd & Wolpin, 2003).

Institutional productivity also depends on how resources are utilised within different organisational and social contexts. Factors such as school governance, incentive structures, teacher motivation, and organisational culture significantly influence educational outcomes. Recent studies in institutional economics and organisational theory show that institutions with similar resource levels may achieve very different results due

to differences in management and governance practices (Aghion et al., 2010; Hanushek et al., 2017). Thus, educational productivity depends not only on resource availability but also on effective institutional management.

Overall, it should be conceptualized broadly, encompassing not only input–output efficiency but also quality, accountability, and institutional effectiveness. A broader definition of education productivity provides the foundation for education production function analysis and motivates the diverse modelling approaches reviewed in subsequent sections. The multidimensional, cumulative, and institutionally rooted nature of education production is essential for developing meaningful productivity measures and for deriving realistic policy conclusions from empirical studies.

Education Production Function Models

The education production functions/models formally represent the process of transforming educational inputs into learning outcomes. Microeconomic production theory conceptualises the EPF as the technology that maps a set of inputs into a set of outputs. After the pioneering work of Becker (1962), Ben-Porath (1967), and Mincer (1958) on human capital theory, empirical research in the economics of education focused on the EPFs as the significant analytical framework.



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General Education Production Function

Equation (1) illustrates the most general form of an education production function for individual i in institution j at time t .

$$A_{ijt} = f(F_{it}, P_{jt}, S_{jt}, T_{jt}, \mu_i) \dots \dots \dots (1)$$

Where A_{ijt} denotes educational achievements, F_{it} represents family and background, characteristics, P_{jt} captures peer and community effects, S_{jt} denotes school or institutional resources, T_{jt} represents teacher and instructional inputs, and μ_i reflects unobserved innate ability.

Though the formulation of equation (1) highlights the multidimensional nature of education production, there is an intrinsic difficulty in the formulation of EPF in terms of isolating causal effects when key components, particularly innate ability and historical inputs, are unobserved (Hanushek, 1986; Todd & Wolpin, 2003). However, this formulation provided the three developments of educational production functions in empirical literature, including contemporaneous, value-added, and linear growth models.

Contemporaneous Education Production Model

The contemporaneous EPF strictly models the current educational outcomes on current inputs as illustrated in equation (2).

$$A_{ijt} = \alpha + \beta X_{ijt} + \varepsilon_{ijt} \dots \dots \dots (2)$$

Where X_{ijt} is a vector of contemporaneous education inputs, and ε_{ijt} is a composite error term. Coleman et al. (1966) use this model specification for estimating the educational production function, and thereafter, this specification has extensive empirical applications in early cross-sectional studies. However, the specification includes assumptions that there are no effects of past inputs and innate ability on the current achievement, or they are uncorrelated (Todd & Wolpin, 2003). These assumptions are rarely defensible in practice. Recent empirical work confirms that contemporaneous models are highly sensitive to omitted variable bias. Therefore, contemporaneous models should be interpreted as descriptive rather than causal representations of education productivity (Gibbons et al., 2013).

Value-Added Education Production Model

Value-added models extend the contemporaneous framework by explicitly accounting for prior achievement, and the general model specification is given in equation (3).

$$A_{ijt} = \alpha + \rho A_{ijt-1} + \beta X_{ijt} + \varepsilon_{ijt} \dots \dots \dots (3)$$

Where A_{ijt-1} denotes lagged achievement. Unobserved ability and cumulative historical inputs are proxied by prior achievement offering a more reliable representation of the learning process. The β coefficient can be interpreted as the marginal contribution of current inputs to achievement growth conditional on



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priori achievement, under appropriate assumptions regarding input decay and error structure (Hanushek & Rivkin, 2010). Value-added models have been extensively applied in productivity and accountability studies, particularly in evaluating teachers and school effectiveness.

Linear-Growth Education Production Model

This alternative approach focuses directly on achievement gains rather than levels as represented in equation (4).

$$\Delta A_{ijt} = A_{ijt} - A_{ijt-1} = \alpha + \beta X_{ijt} + u_{ijt} \dots \dots (4)$$

The linear growth specification aims to isolate time-invariant unobserved ability when baseline achievement data are available, but long achievement histories are absent. It assumes homogeneous learning trajectories and constant effects of past inputs over time. However, Ding and Lehrer (2014) show that these assumptions are often violated in heterogeneous education systems, limiting interpretability.

Comparative Perspective and Recent Extensions

EPF specifications involve trade-offs among data requirements, identifying assumptions, and interpretability. Consequently, recent research increasingly adopts hybrid approaches combining value-added models with panel data or multilevel frameworks to better control for unobserved heterogeneity (Lavy, 2015; Rivkin, Hanushek, & Kain, 2005). Modern EPF

approaches, including stochastic frontier models, distinguish technical inefficiency from random noise and evaluate relative institutional productivity (Agasisti & Johnes, 2015; De Witte & López-Torres, 2017). These developments highlight the importance of modelling learning dynamics alongside efficiency differences.

Methodological Approaches to Measuring Productivity in Education

The methodological approach used to estimate the education production function strongly influences productivity measurement in education. Although the EPF framework provides a useful theoretical basis, empirical implementation is complicated by data limitations, unobserved heterogeneity, endogeneity, and the multidimensional nature of educational outputs (Hanushek, 1979; Todd & Wolpin, 2003; Sullivan et al., 2012). Consequently, researchers have employed various econometric and efficiency-based methods, each with different assumptions and inferential implications (Aigner et al., 1977; De Witte & López-Torres, 2017; Ferro, 2020). Existing methodologies are generally classified into two categories, namely regression-based approaches and frontier-based efficiency models. The following section critically reviews these methods, focusing on their conceptual foundations, formal structures, and major limitations.



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Regression-Based Approaches

Ordinary Least Squares and Related Models

Regression analysis is one of the most widely used empirical approach in education production studies particularly using ordinary least squares (OLS). The simplest form of the model is illustrated in equation (5).

$$A_{ijt} = \alpha + \beta X_{ijt} + \varepsilon_{ijt} \dots \dots \dots (5)$$

Where A_{ijt} denotes educational achievement and X_{ijt} represents a vector of education inputs.

OLS-based education production functions assume that all production units operate on the same production frontier and estimate the average relationship between educational inputs and outputs. Although widely used, these models have important limitations for productivity analysis because they cannot distinguish technical inefficiency from random error (Hanushek, 1986). Endogeneity is another major concern, as educational outcomes are jointly influenced by parental choice, school selection, and policy responses to performance (Hanushek, 1979; Todd & Wolpin, 2003). Formally, endogeneity occurs when explanatory variables are correlated with the error term;

$$\text{Cov}(X_{ijt}, \varepsilon_{ijt}) \neq 0 \dots \dots \dots (6)$$

Equation (6) provide biased and inconsistent parameter estimates. Empirical studies consistently show that the major drawback of empirical

studies is failure to address endogeneity providing substantially mixed findings in the EPF literature (Levačić & Vignoles, 2002; Todd & Wolpin, 2003).

Instrumental Variables and Panel Data Models

Researchers have used instrumental variable (IV) and panel data techniques to reduce endogeneity problems in education production functions. The IV approach replaces endogenous regressors with instruments correlated with inputs but uncorrelated with the error term. However, the reliability of IV estimates depends heavily on the availability of valid and strong instruments, which remains a major challenge in education research (Angrist & Lavy, 1999; Hanushek, 1979; Todd & Wolpin, 2003). Panel data models provide an alternative by exploiting within-unit variation over time. A simple fixed-effects specification is presented in Equation (7).

$$A_{ijt} = \alpha_i + \beta X_{ijt} + u_{ijt} \dots \dots \dots (7)$$

$$\text{Cov}(X_{ijt}, \varepsilon_{ijt}) \neq 0$$

Where X_{ijt} represents observed educational inputs, u_{ijt} denotes time-varying idiosyncratic errors, and α_i captures time-invariant unobserved heterogeneity such as innate ability or institutional characteristics. Panel data approaches are increasingly used in productivity analysis because they control for unobserved heterogeneity



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without relying on external instruments, particularly in higher education contexts (Kingdon, 2006). However, empirical studies show that panel models may still produce biased estimates when unobserved characteristics vary over time or when measurement errors are substantial (Dearden et al., 2014).

Frontier-Based Efficiency Models

Frontier methods were introduced to overcome the limitations of regression-based approaches that estimate only average production relationships. By explicitly modelling technical efficiency, frontier methods are highly suitable for educational productivity analysis. The literature identifies two main frontier approaches: parametric methods, particularly SFA, and nonparametric methods such as DEA.

Stochastic Frontier Analysis (SFA)

Stochastic frontier analysis is an extension of the standard regression framework, which allows to decompose the error term into random noise and inefficiency components (Aigner et al., 1977; Meeusen & van den Broeck, 1977). A classic stochastic frontier EPF can be written as in equation (8).

$$A_{ijt} = f(X_{ijt}; \beta) + v_{ijt} - u_{ijt} \dots \dots \dots (8)$$

Where v_{ijt} represents symmetric random noise, and $u_{ijt} \geq 0$ captures technical inefficiency. Unlike standard regression ($A_{ijt} = \alpha + \beta X_{ijt} + \epsilon_{ijt}$), SFA splits the residual into two parts:

random noise v_{ijt} and inefficiency u_{ijt} . This allows measuring institution-specific efficiency.

Using the decomposition in Equation (8), researchers can estimate both the production frontier and unit-specific efficiency scores. SFA is particularly attractive in education because it separates inefficiency from measurement error and random shocks (Aigner et al., 1977; Ferro, 2020; Kumbhakar, Wang, & Horncastle, 2015). Recent studies have extended SFA to panel data models with dynamic inefficiency, enabling analysis of school and teacher performance over time (Agasisti & Johnes, 2015; Kumbhakar, Wang, & Horncastle, 2015). However, SFA relies on strong parametric assumptions regarding the production function and inefficiency distribution. Violations of these assumptions can substantially distort efficiency estimates and reduce comparability across studies (De Witte & López-Torres, 2017).

Data Envelopment Analysis (DEA)

Data Envelopment Analysis provides a non-parametric alternative to SFA. The DEA production frontier entirely depends on linear programming techniques. Therefore, no functional forms are required for the production frontier. The general input-oriented DEA model can be expressed as in equation (9).

$$\min_{\theta, \lambda} \theta \text{ s.t. } Y\lambda \geq y_i, ST. \theta X_i \geq X\lambda, \lambda \geq 0 \dots (9)$$



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Where θ represents the efficiency score. DEA is the most appropriate approach when multiple inputs and outputs are available in education systems. Therefore, DEA has wide applications in efficiency analysis, including the areas of schools, universities, and education systems across countries (Charnes et al., 1978; De Witte & López-Torres, 2017; Ferro, 2020). However, the DEA assigns all deviations from the frontier to inefficiency. Therefore, estimates are highly sensitive to measurement error and outliers. Some methodological approaches, including bootstrapping techniques, attempted to address these limitations, providing statistical inference for efficiency estimates (Daraio & Simar, 2014; Simar & Wilson, 2007).

Functional Form and Model Specification

The choice of functional form in EPF models presents a major methodological challenge. Linear and log-linear (Cobb–Douglas) specifications are widely used because of their simplicity, but they often fail to capture nonlinear relationships between educational inputs and outputs (Figlio, 1999; Griliches & Mairesse, 2010; Hanushek, 1979). Consequently, recent studies increasingly employ flexible functional forms and extensive robustness testing to examine the sensitivity of productivity estimates, especially where complex interactions and nonlinear effects exist (Eom, 2022; Wedel, 2021). Existing literature

identifies model misspecification as a major reason for inconsistent empirical findings in education production function studies (De Witte & Mergoni, 2023; Figlio, 1999).

Methodological Implications for Productivity Measurement

This review highlights that conclusions about educational productivity depend heavily on methodological choice. Regression approaches are useful for identifying relationships between inputs and achievement, but they are limited in measuring production efficiency. In contrast, frontier methods directly estimate efficiency but require stronger assumptions and higher-quality data (Aigner et al., 1977; De Witte & López-Torres, 2017; Hanushek, 1986). Consequently, recent research increasingly supports integrated approaches combining EPF models with efficiency analysis, panel data, and multi-output frameworks to generate more reliable productivity estimates (Ahmad, 2024; Lou, 2024). These approaches are particularly valuable for higher education systems in developing countries, where policy decisions must be made under significant resource constraints.

Emerging and Hybrid Approaches in Education Productivity Analysis

Recent developments in EPF research increasingly move beyond traditional regression and frontier methods toward integrated, flexible, and data-intensive approaches. These models address major limitations related to

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endogeneity, rigid functional forms, hierarchical data structures, and dynamic learning processes, which often create empirical inconsistencies in productivity analysis.

A key development is the use of dynamic panel and multi-period production models that incorporate learning persistence and input decay. Extending the cumulative learning framework of Todd and Wolpin (2003), recent studies allow for nonlinear decay of past inputs and heterogeneous persistence effects, providing a more realistic representation of the education production process (Andrabi et al., 2011; Cunha et al., 2010; Rothstein, 2010). These models are commonly estimated using dynamic panel techniques such as system GMM, which reduce endogeneity problems caused by simultaneity and omitted variables (Arellano & Bover, 1995; Blundell & Bond, 1998). Such approaches are especially relevant in higher education, where productivity evolves through accumulated institutional investments.

Another major development is the use of hierarchical or multilevel production function models that account for the nested structure of education data, such as students within classes and institutions. Traditional EPF models often ignore this structure, potentially biasing productivity estimates. Multilevel models allow random coefficients and cross-level interactions, helping distinguish within- and between-institution

productivity differences (Goldstein, 2011; Raudenbush & Bryk, 2002). In higher education, these models are increasingly applied to examine how institutional characteristics influence the effectiveness of teaching quality, peer composition, and resource allocation.

Recent literature highlights the increasing use of machine learning (ML) techniques in education productivity analysis. Methods such as random forests, gradient boosting, and neural networks provide flexible, nonparametric alternatives capable of capturing complex nonlinearities and high-dimensional interactions among educational inputs (Athey & Imbens, 2019; Mullainathan & Spiess, 2017). The integration of ML with causal approaches, including double/debiased machine learning, has improved its usefulness for estimating causal effects (Chernozhukov et al., 2018). These methods are especially valuable when analysing large administrative datasets where traditional parametric models may be too restrictive.

Bayesian methods to education production and efficiency analysis is some other recent development in the context. Bayesian hierarchical models incorporate prior information, model parameter uncertainty, and estimate complex multilevel structures (Gelman et al., 2013). Bayesian stochastic frontier models also allow flexible inefficiency specifications and probabilistic interpretation of efficiency scores, particularly useful in education

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settings with small samples, measurement error, and latent heterogeneity (Kumbhakar & Tsionas, 2011).

Recent research increasingly integrates causal inference methods with education production function models to obtain more credible productivity estimates. Techniques such as regression discontinuity, difference-in-differences, and instrumental variables are embedded within EPF frameworks to improve causal interpretation (Angrist & Pischke, 2009; Chetty et al., 2014). More advanced studies combine EPF models with structural and semi-structural causal frameworks to analyse policy counterfactuals and dynamic treatment effects. Overall, these developments reflect a shift toward hybrid approaches that combine econometric rigor, computational flexibility, and richer data structures. By integrating dynamic modelling, hierarchical structures, causal inference, and machine learning, recent EPF research provides more robust and policy-relevant analyses of education productivity and efficiency.

Joint Production and Multiple Outputs in Education

A major limitation of education production function literature is the treatment of education as a single-output process. Traditional studies mainly focus on cognitive outcomes such as test scores and examination results because they are easily

measurable and comparable (Hanushek, 1979; Summers & Wolfe, 1977). However, education simultaneously produces multiple outcomes, including cognitive skills, non-cognitive behaviours, attitudes, and labour-market competencies. Ignoring this multidimensional nature can lead to misleading productivity interpretations (Chizmar & Zak, 1983; Heckman & Kautz, 2012; OECD, 2018). Although alternative approaches such as canonical joint-production models and multi-output efficiency methods have been proposed to address this issue, their empirical application remains limited (Agasisti & Johnes, 2015; Ranasinghe, 1999; Vinod, 1968).

The existence of multiple educational outputs raises important questions about how inputs are transformed into outcomes. Rao (1969) distinguishes between multiple production and joint production. In multiple production, outputs are generated through separate activities competing for limited inputs, whereas in joint production, a common set of inputs simultaneously generates several outputs with little input exhaustion (Chizmar & Zak, 1983; Rao, 1969). Education is widely considered a joint production process because inputs such as teacher effort, classroom instruction, institutional resources, and peer interactions jointly produce cognitive achievement, skills formation, and behavioural outcomes (Hanushek, 1979; Chizmar & McCarney, 1984). Consequently, single-output models overlook



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important complementarities and trade-offs among educational outcomes. Early EPF studies commonly estimated separate single-output equations, a practice later criticised by Hanushek (1986) as a major weakness of the literature. Therefore, formal joint-production frameworks are essential for more accurate productivity analysis.

Joint Education Production Function: Conceptual Framework

The general joint education production function can be given as in equation (10).

$$(A_{1i}, A_{2i}, \dots, A_{ki}) = f(X_i) \dots \dots \dots (10)$$

Where A_{ki} denotes the k -th educational output for an individual or institution i , and X_i represents shared inputs. This formulation implies that inputs such as instructional time, teacher quality, and peer interactions simultaneously influence multiple educational outcomes. Unlike single-output models, joint production frameworks recognise potential complementarities and trade-offs among outcomes. Early studies by Vinod (1968) and Rao (1969) introduced canonical correlation techniques for modelling joint production, later adapted to education by Chizmar and Zak (1983) and Chizmar and McCarney (1984). These studies showed that joint production models provide deeper insights into education processes than separate regression equations. The approach is especially relevant in higher education,

where institutions simultaneously pursue teaching quality, research productivity, and graduate employability. More recent multi-output frontier and distance-function approaches extend this framework with greater flexibility and policy relevance (Agasisti, 2020; Daraio & Simar, 2014).

Functional Forms for Joint Education Production

The generalised Cobb–Douglas specification is the widely used functional form for joint EPFs in education. The joint production function, assuming two outcomes, can be written as equation (11).

$$A_{1i}^{\alpha_1} A_{2i}^{\alpha_2} = g(X_i) \dots \dots \dots (11)$$

or, equivalently in logarithmic form is given as in equation (12).

$$\alpha_1 \ln A_{1i} + \alpha_2 \ln A_{2i} = \beta_0 + \sum_j \beta_j \ln X_{ji} + \varepsilon_i \dots (12)$$

The logarithmic joint-production formulation enables estimation of input elasticities while preserving the multidimensional nature of educational outputs. Unlike simultaneous equation models that impose causal relationships among outputs, joint EPFs treat outcomes as jointly produced within a single process. Chizmar and Zak (1983) applied this approach to model both cognitive achievement and student attitudes using canonical regression methods, demonstrating results that differed substantially from separate

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estimations. In higher education, joint production frameworks are increasingly used to analyse teaching, research, and labour-market outcomes simultaneously (Agasisti & Johnes, 2015; De Witte & López-Torres, 2017). Empirical evidence shows that institutions may perform efficiently in some outputs while underperforming in others, highlighting trade-offs that single-output models fail to capture (Lou, 2024; Sullivan et al., 2012).

Canonical Regression and Multi-Output Estimation

Canonical correlation and canonical regression methods allow education systems with multiple inputs and outputs to be modelled within a joint-production framework. These approaches maximise the relationship between linear combinations of inputs and outputs, recognising the multidimensional nature of educational production (Chizmar & Zak, 1983; Rao, 1969; Vinod, 1968). Ranasinghe (1999) applied canonical regression in Sri Lanka to measure education quality as a latent construct derived from observable inputs, enabling comparison of school productivity without relying on a single output indicator. This approach is particularly useful when educational outputs are multidimensional and difficult to observe directly. More recent studies integrate joint-production concepts with frontier efficiency techniques, including multi-output DEA and distance-function approaches, which evaluate efficiency

without requiring price information (Agasisti, 2020; Bhutoria, 2022; Daraio & Simar, 2014).

Implications for Productivity Measurement

Joint production approaches have several important applications in education productivity analysis. First, they provide a more comprehensive assessment of education systems that pursue multiple objectives simultaneously. Second, they help policymakers identify trade-offs among different educational outcomes. Third, they reduce bias arising from omitted output dimensions. These advantages have increased interest in joint-production frameworks, particularly in higher education systems emphasising multidimensional performance and performance-based funding (Agasisti, 2020; OECD, 2018). Consequently, joint EPF models are increasingly relevant for modern education policy analysis. Despite their advantages, empirical applications remain limited because of data constraints and computational complexity. Nevertheless, these approaches offer strong potential for analysing education productivity, especially in developing countries and public higher education systems where multiple outputs are central.

Synthesis of Empirical Evidence from Education Production Studies

Empirical evidence from education production function studies is multi-dimensional, including diverse



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education systems, levels, and methodological approaches for several decades. However, the findings are inclusive regarding the magnitude or even the direction of the effects of educational inputs on outcomes (Hanushek, 1986, 1997). Accordingly, this section critically analyses the empirical findings, specifically focusing on how output selection, input specification, and methodological choices influence productivity results.

Education Outputs: Measurement and Empirical Implications

Most education production function studies use cognitive achievement measures such as standardised test scores, examination results, and grade point averages as proxies for educational output (Bishop, 1991; Hanushek, 1979). These indicators are widely used because they are observable, comparable across institutions, and strongly linked to labour-market outcomes (Hanushek, 1979; Summers & Wolfe, 1977). Contemporary studies continue to rely on cognitive measures because they provide consistent and policy-relevant benchmarks for evaluating educational performance (Chetty et al., 2014; OECD, 2018).

However, recent research argues that exclusive reliance on cognitive outcomes understates the full productivity of education systems because it ignores non-cognitive skills and long-term outcomes (Heckman &

Kautz, 2012). Empirical evidence also shows that productivity estimates are highly sensitive to how educational outputs are defined (De Witte & Mergoni, 2023; Figlio, 1999). Narrow cognitive measures often produce weak or inconsistent estimates due to measurement limitations and endogeneity problems (Hanushek, 1986). In contrast, studies using broader outcomes such as graduation rates, degree completion, and labour-market returns report stronger relationships between educational inputs and performance (Card & Krueger, 1996; Hanushek & Woessmann, 2015). Recent longitudinal and administrative-data studies further demonstrate that education productivity is better reflected through long-term economic and social outcomes rather than short-term test-score gains alone (Chetty et al., 2014; Deming, 2017; Heckman, & Mosso, 2014).

Student and Family Background Inputs

Empirical findings by Coleman et al. (1966) and Hanushek (1986) identify family background factors such as parental education, socio-economic status, and home learning environments as major determinants of educational outcomes. Recent longitudinal studies confirm that these influences have cumulative effects on both cognitive and non-cognitive skill formation, shaping long-term educational attainment and labour-market performance (Chetty et al.,

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2014; Cunha et al., 2010; OECD, 2018). The Coleman Report was historically important in highlighting the strong influence of family background relative to school resources. However, later research shows that the relative importance of family inputs varies with model specification and data structure (Bishop, 1991; Card & Krueger, 1996). Panel and value-added models indicate that much of this influence operates through early learning accumulation, emphasising the importance of dynamic modelling and early-life educational investments (Carneiro & Heckman, 2021; Rothstein, 2010).

School Resources and Instructional Inputs

Empirical evidence on school resources such as class size, per-student expenditure, teacher qualifications, and instructional time remains mixed in productivity analyses (Card & Krueger, 1996; Hanushek, 1986). Early cross-sectional EPF studies often found weak or statistically insignificant relationships between school resources and student achievement.

Later studies using quasi-experimental methods, panel data, and value-added models show that the effects of school resources are highly context dependent. Evidence suggests that smaller class sizes particularly benefit disadvantaged students, while effects for other groups are more limited or mixed (Angrist & Lavy, 1999; Krueger, 2003). Similarly, instructional time and student attendance become significant

productivity factors when endogeneity and selection bias are properly addressed (Lavy, 2015; Romer, 1993; Stanca, 2006). Frontier-based studies further indicate that performance differences often arise from variations in how efficiently institutions use resources rather than from resource levels alone. Consequently, institutional management, organisational practices, and governance structures strongly influence educational productivity (Agasisti & Johnes, 2015). Recent DEA, SFA, and Malmquist studies confirm persistent efficiency differences across schools and higher education institutions due to technical and managerial factors (Ahmad, 2024; Ferro, 2020).

Peer Effects and Classroom Composition

Peer effects are among the most challenging yet important factors in education production studies. Existing research shows that higher-achieving peers can positively influence individual academic performance, making peer composition an important educational input beyond formal school resources (Sacerdote, 2001; Zimmer & Toma, 2000). However, measuring peer effects is difficult because students simultaneously influence and are influenced by their peers, creating endogeneity problems (Manski, 1993). As a result, studies that fail to address endogenous peer selection often overestimate peer effects (Burke & Sass, 2013; Carrell et



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al., 2013). Research using random assignment, natural experiments, and panel-data methods generally finds smaller but more credible peer effects (Carrell et al., 2009). Evidence also indicates that peer effects are heterogeneous and context-dependent, with stronger impacts on disadvantaged or lower-ability students than on high-ability students (Lavy, Paserman, & Schlosser, 2012). Recent administrative-data studies further show that peer influence affects long-term outcomes such as course selection, persistence, and graduation, highlighting its importance for broader productivity analysis (Chetty et al., 2014).

Methodological Sources of Empirical Ambiguity

The education production function literature shows that methodological choices are a major source of inconsistent findings on educational productivity. Differences in model specification, data structure, outcome definitions, and estimation methods can produce very different conclusions about the effectiveness of educational inputs, even when studies analyse similar contexts and datasets (Hanushek, 1986; Todd & Wolpin, 2003).

Most EPF models estimated using regression techniques, especially cross-sectional methods, identify average input-output relationships but cannot separate technical inefficiency from random variation (Aigner et al., 1977;

Hanushek, 1986). Consequently, estimated coefficients may combine inefficiency, omitted variables, and measurement errors, contributing to inconsistent findings in early EPF studies (Todd & Wolpin, 2003). Although panel data and value-added models improve estimation by controlling for unobserved heterogeneity and prior achievement, they remain sensitive to assumptions regarding learning persistence and input decay over time (Rothstein, 2010). Another major source of ambiguity is the choice of functional form. Linear and log-linear specifications, though convenient, may conceal nonlinearities and interaction effects (Figlio, 1999). Therefore, recent literature increasingly emphasises flexible modelling strategies, specification sensitivity analysis, and systematic robustness testing to improve the reliability of education productivity estimates (Agasisti, 2020; OECD, 2018).

Frontier-based efficiency measuring techniques is an alternative approach to illustrate how methodological choice shapes empirical conclusions. Compared to regression-based approaches, stochastic frontier analysis and data envelopment analysis explicitly separate inefficiency component from statistical noise, empowering comparisons of relative performance across institutions (Aigner et al., 1977; Charnes et al., 1978; De Witte & López-Torres, 2017). Frontier studies explicitly reveal that observed performance differences are

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mainly due to the variation in the efficiency of resource use rather than differences in resource availability. Therefore, Frontier studies emphasize the importance of institutional practices, governance, and management quality (Agasisti & Johnes, 2015; De Witte & López-Torres, 2017).

Recent applications of panel frontier models, conditional efficiency frameworks, and Malmquist productivity indices reveal persistent technical inefficiencies across education systems and highlight the role of organisational and contextual factors in shaping productivity outcomes (Ahmad, 2024; Daraio & Simar, 2014). Methodological ambiguity also emerges when educational outputs are narrowly defined or when contextual variables such as peer composition and student sorting are omitted. Ignoring these factors may wrongly attribute performance differences to institutional inefficiency rather than compositional effects (De Witte & López-Torres, 2017; Manski, 1993). Overall, the literature identifies methodological heterogeneity as a major source of inconsistent findings, emphasising careful model selection, transparent assumptions, and integrated analytical approaches.

Summary of Empirical Insights

The review of education production and efficiency literature highlights several important conclusions. First,

productivity estimates are highly sensitive to output definitions, data structures, and model specifications. Narrow and short-term cognitive measures often produce weak or inconsistent findings, whereas broader and long-term outcomes such as completion rates and labour-market performance reveal stronger productivity effects.

Second, early-life investments and cumulative learning processes play a crucial role in shaping educational outcomes through family and student background characteristics. Although panel and value-added models reduce contemporaneous family effects, they do not eliminate their long-run influence on productivity. Third, the effects of school resources and peer influences depend heavily on institutional context and heterogeneity. Productivity differences often reflect institutional practices, peer composition, and efficient resource utilisation rather than resource levels alone.

Finally, inconsistencies in EPF findings largely arise from methodological heterogeneity rather than differences in educational processes. Therefore, dynamic, multi-output, and efficiency-oriented frameworks are essential for more reliable productivity analysis across diverse educational settings. To enhance comparability across studies, Table 1 synthesises the core features of representative EPF research, including modelling approaches, data structures, and key results. The table illustrates



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how differences in methodology and output specification contribute to divergent findings and highlights areas

where further integration, particularly across efficiency and multi-output frameworks, is needed.

Table 1. Summary of Key Approaches in Education Production Function (EPF) Literature

Author	Model / Method / Equation	Data Context	Key Inputs	Key Outputs	Main Findings / Results
Coleman et al. (1966)	Contemporaneous EPF $A_{ijt} = \alpha + \beta X_{ijt} + e_{ijt}$	US, Primary schools, Cross-sectional	Family background, school resources, peers	Test scores	Family background has stronger influence than school resources
Hanushek (1979, 1986)	OLS-based EPF	Various cross-sectional datasets	School inputs, teacher characteristics	Test scores	Mixed and inconsistent effects of school resources; methodological sensitivity
Todd & Wolpin (2003)	Dynamic / Cumulative EPF	Panel / longitudinal data	Current & past inputs, family background	Achievement	Learning is cumulative; omission of past inputs biases estimates
Hanushek & Rivkin (2010)	Value-added model $A_{ijt} = \alpha + A_{ijt-1} + \beta X_{ijt} + e_{ijt}$	Panel data (US schools)	Current inputs, prior achievement	Achievement gains	Controls for unobserved ability; improved causal interpretation
Rothstein (2010)	Value-added with decay dynamics	Panel education data	Teacher inputs, lagged achievement	Test scores	Value-added estimates sensitive to assumptions on persistence
Ding & Lehrer (2014)	Linear growth model $\Delta A_{ijt} = \alpha + \beta X_{ijt} + u_{ijt}$	Experimental / panel data	Instructional inputs	Achievement gains	Growth models may oversimplify learning dynamics
Angrist & Lavy (1999)	IV / quasi-experimental EPF	Israel, natural experiment	Class size	Test scores	Smaller class size improves outcomes (causal evidence)
Chetty et al. (2014)	Value-added with causal inference	Administrative panel data	Teacher quality	Long-term outcomes	Teacher quality has long-term



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					economic impacts
Aigner et al. (1977)	Stochastic Frontier Analysis (SFA) $A = f(X) + v - u$	Cross-sectional / panel	School resources	Achievement	Separates inefficiency from noise; measures technical efficiency
Agasisti & Johnes (2015)	Panel SFA / efficiency models	Higher education (US)	Institutional resources	Graduation, research outputs	Efficiency differences explain performance gaps
Charnes et al. (1978)	Data Envelopment Analysis (DEA)	Multi-input/multi-output systems	Multiple inputs	Multiple outputs	Non-parametric efficiency measurement; sensitive to outliers
Daraio & Simar (2014)	Robust / conditional DEA	Cross-country education systems	Environmental variables	Multiple outputs	Incorporating context improves efficiency estimates
Chizmar & Zak (1983)	Joint production function	University-level data	Shared educational inputs	Cognitive & attitudinal outcomes	Outputs are jointly produced; single-output models are biased
Ranasinghe (1999)	Canonical regression (joint EPF)	Sri Lanka schools	Educational inputs	Latent education quality	Captures multidimensional outputs effectively
Cunha et al. (2010)	Structural dynamic skill formation model	Longitudinal data	Early-life and later inputs	Cognitive & non-cognitive skills	Strong dynamic complementarity in skill formation
Raudenbush & Bryk (2002)	Multilevel / hierarchical models	Nested education data	Student & school-level inputs	Achievement	Separates within- and between-school effects
Athey & Imbens (2019)	Machine learning EPF extensions	Large datasets	High-dimensional inputs	Various outcomes	Captures non-linearities and complex interactions
Chernozhukov et al. (2018)	Double machine learning (causal ML)	High-dimensional panel data	Endogenous inputs	Causal effects	Improves causal identification in complex EPFs



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Kumbhakar & Tsionas (2011)	Bayesian SFA	Small / noisy datasets	Institutional inputs	Efficiency scores	Flexible modelling of inefficiency and uncertainty
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Note. The content of the table was constructed by author based on available literature

Policy Implications and Directions for Future Research

The studies on education production function analysed in previous sections conclude that education productivity is a multi-dimensional term that represents how much education is being produced with a given resource, but also how resources are employed effectively based on the institution's and society's goals. The inconclusive findings in literature does not imply the insignificance of education productivity. It explicitly emphasizes using improved analytical frameworks to better hance policy interpretations. This section focuses on key policy implications arising from education production function studies and outlines directions for future research aimed at strengthening productivity analysis in education.

Policy Implications

First, mixed findings on resource effects challenge purely input-driven education policies. Although investments in class size reduction, instructional time, and teacher quality can improve outcomes, especially for disadvantaged students, their effectiveness depends heavily on institutional and contextual conditions.

Simply increasing resources without improving efficiency, governance, and incentives is unlikely to generate substantial productivity gains (Agasisti & Johnes, 2015; Hanushek, 2003).

Second, frontier-based and multi-output frameworks highlight the importance of efficiency-oriented policies. Empirical evidence suggests that differences in institutional performance often arise from how efficiently resources are utilised rather than from resource availability alone. This underscores the policy relevance of institutional governance, management quality, and accountability systems in improving educational productivity (Agasisti, 2020; De Witte & López-Torres, 2017).

Third, the multidimensional nature of educational outcomes has important implications for performance measurement and accountability. Reliance solely on cognitive indicators may misrepresent institutional performance. Therefore, policymakers should adopt multidimensional productivity frameworks incorporating cognitive, non-cognitive, and labour-market outcomes, particularly in higher education and skill-oriented systems (Sun, 2023; OECD, 2018).

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Finally, productivity-enhancing policies should be context-sensitive and long-term oriented. Early-life learning environments, peer composition, and socio-economic disadvantages strongly influence later educational productivity. Such targeted interventions are especially important in developing countries, where resource constraints directly affect efficiency and cumulative learning outcomes.

Directions for Future Research

This review directs several future research. The future research needs to adopt analytical frameworks that combine both education production function modelling and efficiency analysis. These frameworks should incorporate panel data methods and multi-output approaches. Such integration will produce more convincing productivity estimates across heterogeneous education systems, improving identification and reducing the specification bias in production functions/frontiers.

Second, future research should focus on obtaining improved measurements of educational outcomes. This requires clear expansion of test scores to include non-cognitive skills, persistence, employability, and longer-term outcomes. This fulfilment would provide a more comprehensive assessment of education productivity, directly aligning empirical assessment more intently with policy objectives.

Third, since the cumulative learning significantly contributes to the educational productivity, major emphasis should be focused on dynamic modelling frameworks capturing the cumulative nature of learning. Longitudinal data and life-cycle approaches are recommended in this context, since they help to understand how early inputs interact with later institutional environments in shaping long-term productivity outcomes.

Finally, since governance structures, managerial practices, and incentive systems influence efficiency, future studies can lead to more benefits by closely following institutional and organisational theory, bridging the gap between productivity measurement and actionable policy designs.

CONCLUSION

The study critically reviewed EPF research with particular emphasis on productivity measurement in education. Decades of empirical and theoretical work reveal considerable ambiguity in findings, not because productivity analysis lacks relevance, but because education is inherently difficult to model as a production process. Many inconsistent findings arise from conceptual, methodological, and measurement challenges. The review shows that early EPF studies relied heavily on contemporaneous single-output regression models, which often produced conflicting



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conclusions regarding the effectiveness of educational inputs. Subsequent methodological developments, including value-added models, growth specifications, panel data methods, and frontier efficiency approaches, have improved analytical rigor. However, no single framework fully resolves the challenges of productivity analysis. Productivity estimates remain highly sensitive to assumptions regarding learning dynamics, functional forms, endogeneity, and unobserved heterogeneity. A major contribution of this review is its emphasis on the multidimensional nature of educational outputs. The study highlights the limitations of narrowly defined productivity measures and underscores the importance of joint-production and multi-output frameworks capable of capturing complementarities and trade-offs among educational outcomes. The review also demonstrates the value of integrating efficiency analysis into EPF models to explain why institutions with similar resource endowments often achieve different performance outcomes.

From a policy perspective, the findings reinforce the importance of efficiency-oriented and context-sensitive education policies rather than simplistic input-driven reforms. Productivity improvements depend not only on additional resources but also on effective governance, institutional innovation, and efficient resource utilisation. These issues are

particularly important in higher education and developing-country contexts where resource constraints intensify the need for efficient allocation. The review further identifies several priorities for future research, including the integration of dynamic production models with efficiency analysis, expansion of joint-production frameworks, improved measurement of multidimensional educational outcomes, and stronger theoretical foundations for empirical modelling. Better data quality and advanced analytical methods will be essential for addressing these challenges. Overall, the review concludes that education productivity measurement is complex but essential. Advancing the field requires rigorous methodological frameworks that recognise the cumulative, multidimensional, and institutionally embedded nature of education production, thereby improving both academic understanding and evidence-based education policy.

References

- Agasisti, T. (2020). Performance-based funding for higher education: A critical review of the literature. *Journal of Higher Education Policy and Management*, 42(6), 589–606. <https://doi.org/10.1080/1360080X.2020.1773275>
- Agasisti, T., & Johnes, G. (2015). Efficiency, costs, rankings, and heterogeneity: The case of U.S. higher education. *Studies in Higher Education*, 40(1), 60–82. <https://doi.org/10.1080/03075079.2013.818642>
- Aghion, P., Dewatripont, M., Hoxby, C. M., Mas-Colell, A., & Sapir, A. (2010). The



Original Article

- governance and performance of universities: Evidence from Europe and the US. *Economic Policy*, 25(61), 7–59. <https://doi.org/10.1111/j.1468-0327.2009.00238.x>
- Ahmad, A. M. (2024). Data envelopment analysis in higher education: A systematic review of recent methodological advances. *SAGE Open*, 14(1). <https://doi.org/10.1177/21582440241271153>
- Aigner, D., Lovell, C. A. K., & Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6(1), 21–37. [https://doi.org/10.1016/0304-4076\(77\)90052-5](https://doi.org/10.1016/0304-4076(77)90052-5)
- Andrabi, T., Das, J., Khwaja, A. I., Vishwanath, T., & Zajonc, T. (2011). Do value-added estimates add value? Accounting for learning dynamics. *American Economic Journal: Applied Economics*, 3(3), 29–54. <https://doi.org/10.1257/app.3.3.29>
- Angrist, J. D., & Pischke, J.-S. (2009). *Mostly harmless econometrics: An empiricist's companion*. Princeton University Press.
- Angrist, J. D., & Lavy, V. (1999). Using Maimonides' rule to estimate the effect of class size on scholastic achievement. *Quarterly Journal of Economics*, 114(2), 533–575. <https://doi.org/10.1162/003355399556061>
- Archibald, R. B., & Feldman, D. H. (2018). *Why does college cost so much?* Oxford University Press.
- Arellano, M., & Bover, O. (1995). Another look at the instrumental variable estimation of error-components models. *Journal of Econometrics*, 68(1), 29–51. [https://doi.org/10.1016/0304-4076\(94\)01642-D](https://doi.org/10.1016/0304-4076(94)01642-D)
- Athey, S., & Imbens, G. W. (2019). Machine learning methods that economists should know about. *Annual Review of Economics*, 11, 685–725. <https://doi.org/10.1146/annurev-economics-080217-053433>
- Becker, G. S. (1962). Investment in human capital: A theoretical analysis. *Journal of Political Economy*, 70(5, Pt. 2), 9–49. <https://doi.org/10.1086/258724>
- Belley, P., Frenette, M., & Lochner, L. (2014). Postsecondary attendance by parental income in the U.S. and Canada. *Journal of Human Resources*, 49(2), 413–451. <https://doi.org/10.3368/jhr.49.2.413>
- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy*, 75(4), 352–365. <https://doi.org/10.1086/259291>
- Bhutoria, A. (2022). School efficiency across countries: A data envelopment analysis. *International Journal of Educational Development*, 90, 102573. <https://doi.org/10.1016/j.ijedudev.2022.102573>
- Bishop, J. H. (1991). Achievement, test scores, and relative wages. *Economic Journal*, 101(408), 462–481. <https://doi.org/10.2307/2233551>
- Blundell, R., & Bond, S. (1998). Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics*, 87(1), 115–143. [https://doi.org/10.1016/S0304-4076\(98\)00009-8](https://doi.org/10.1016/S0304-4076(98)00009-8)
- Burke, M. A., & Sass, T. R. (2013). Classroom peer effects and student achievement. *Journal of Labor Economics*, 31(1), 51–82. <https://doi.org/10.1086/666653>
- Card, D., & Krueger, A. B. (1996). School resources and student outcomes: An overview of the literature and new evidence from North and South Carolina. *Journal of Economic Perspectives*, 10(4), 31–50. <https://doi.org/10.1257/jep.10.4.31>
- Carneiro, P., & Heckman, J. J. (2021). Human capital policy. In S. Bradley & C. Green (Eds.), *Handbook of the economics of education* (Vol. 5, pp. 1–112). Elsevier. <https://doi.org/10.1016/bs.heduc.2020.10.001>
- Carrell, S. E., Fullerton, R. L., & West, J. E. (2009). Does your cohort matter? Measuring peer effects in college



Original Article

- achievement. *Journal of Labor Economics*, 27(3), 439–464. <https://doi.org/10.1086/600143>
- Carrell, S. E., Sacerdote, B., & West, J. E. (2013). From natural variation to optimal policy? The Lucas critique meets peer effects. *Econometrica*, 81(3), 855–882. <https://doi.org/10.3982/ECTA10436>
- Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research*, 2(6), 429–444.
- Chernozhukov, V., Chetverikov, D., Demirer, M., Duflo, E., Hansen, C., Newey, W., & Robins, J. (2018). Double/debiased machine learning for treatment and structural parameters. *The Econometrics Journal*, 21(1), C1–C68. <https://doi.org/10.1111/ectj.12097>
- Chetty, R., Friedman, J. N., & Rockoff, J. E. (2014). Measuring the impacts of teachers I: Evaluating bias in teacher value-added estimates. *American Economic Review*, 104(9), 2593–2632. <https://doi.org/10.1257/aer.104.9.2593>
- Chizmar, J. F., & McCarney, J. S. (1984). A joint production model of teaching. *Journal of Economic Education*, 15(3), 246–255. <https://doi.org/10.1080/00220485.1984.10845174>
- Chizmar, J. F., & Zak, T. A. (1983). Canonical estimation of joint production functions. *Econometrica*, 51(1), 37–46. <https://doi.org/10.2307/1912253>
- Coleman, J. S., Campbell, E. Q., Hobson, C. J., McPartland, J., Mood, A. M., Weinfeld, F. D., & York, R. L. (1966). *Equality of educational opportunity*. U.S. Government Printing Office.
- Cunha, F., Heckman, J. J., & Schennach, S. M. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, 78(3), 883–931. <https://doi.org/10.3982/ECTA6551>
- Daraio, C., & Simar, L. (2014). *Advanced robust and nonparametric methods in efficiency analysis*. Springer.
- De Witte, K., & López-Torres, L. (2017). Efficiency in education: A review of literature and a way forward. *Journal of the Operational Research Society*, 68(4), 339–363. <https://doi.org/10.1057/jors.2015.92>
- De Witte, K., & Mergoni, A. (2023). Productivity, efficiency, and management quality in education: A meta-analytic review. *Education Economics*, 31(4), 395–417. <https://doi.org/10.1080/09645292.2023.2182456>
- Dearden, L., Ferri, J., & Meghir, C. (2014). The effect of school quality on educational attainment and wages. *Review of Economics and Statistics*, 96(1), 1–19. https://doi.org/10.1162/REST_a_00395
- Deming, D. J. (2017). The growing importance of social skills in the labor market. *Quarterly Journal of Economics*, 132(4), 1593–1640. <https://doi.org/10.1093/qje/qjx022>
- Ding, W., & Lehrer, S. F. (2014). Estimating treatment effects from contaminated multi-period education experiments: The dynamic impacts of class size reductions. *Journal of Human Resources*, 49(3), 612–651. <https://doi.org/10.3368/jhr.49.3.612>
- Eom, J. (2022). Functional form misspecification and productivity estimation: Evidence from panel data. *Empirical Economics*, 63(2), 709–736.
- Ferro, G. (2020). Higher education efficiency: A systematic review of frontier studies. *Socio-Economic Planning Sciences*, 70, 100730. <https://doi.org/10.1016/j.seps.2019.100730>
- Figlio, D. N. (1999). Functional form and the estimated effects of school resources. *Economics of Education Review*, 18(2), 241–252. [https://doi.org/10.1016/S0272-7757\(98\)00024-2](https://doi.org/10.1016/S0272-7757(98)00024-2)
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian data analysis* (3rd ed.). CRC Press.
- Gibbons, S., Silva, O., & Weinhardt, F. (2013). Everybody needs good neighbours. Evidence from students' outcomes in



Original Article

- England. *Economic Journal*, 123(571), 831–874. <https://doi.org/10.1111/econj.12019>
- Goldstein, H. (2011). *Multilevel statistical models* (4th ed.). Wiley
- Griliches, Z., & Mairesse, J. (2010). Production functions: The search for identification. In *Productivity and growth* (pp. 169–203). Cambridge University Press.
- Hanushek, E. A. (1979). Conceptual and empirical issues in the estimation of educational production functions. *Journal of Human Resources*, 14(3), 351–388. <https://doi.org/10.2307/145575>
- Hanushek, E. A. (1986). The economics of schooling: Production and efficiency in public schools. *Journal of Economic Literature*, 24(3), 1141–1177.
- Hanushek, E. A. (1997). Assessing the effects of school resources on student performance: An update. *Educational Evaluation and Policy Analysis*, 19(2), 141–164. <https://doi.org/10.3102/01623737019002141>
- Hanushek, E. A. (2003). The failure of input-based schooling policies. *Economic Journal*, 113(485), F64–F98. <https://doi.org/10.1111/1468-0297.00099>
- Hanushek, E. A., & Rivkin, S. G. (2010). Generalizations about using value-added measures of teacher quality. *American Economic Review*, 100(2), 267–271. <https://doi.org/10.1257/aer.100.2.267>
- Hanushek, E. A., & Woessmann, L. (2015). *The knowledge capital of nations*. MIT Press.
- Hanushek, E. A., Piopiunik, M., & Wiederhold, S. (2017). The value of smarter teachers: International evidence on teacher cognitive skills and student performance. *Journal of Human Resources*, 54(4), 857–899. <https://doi.org/10.3368/jhr.54.4.0317.8619R1>
- Heckman, J. J., & Kautz, T. (2012). Hard evidence on soft skills. *Labour Economics*, 19(4), 451–464. <https://doi.org/10.1016/j.labeco.2012.05.014>
- Heckman, J. J., & Mosso, S. (2014). The economics of human development and social mobility. *Annual Review of Economics*, 6, 689–733. <https://doi.org/10.1146/annurev-economics-080213-040753>
- Kingdon, G. (2006). Teacher characteristics and student performance in India. *Oxford Bulletin of Economics and Statistics*, 68(5), 545–567.
- Kounetas, K., Anastasiou, A., & Mitropoulos, P. (2023). Educational reforms and secondary school efficiency: Evidence from panel frontier models. *Empirical Economics*, 65(3), 1247–1278. <https://doi.org/10.1007/s00181-022-02265-8>
- Krueger, A. B. (2003). Economic considerations and class size. *Economic Journal*, 113(485), F34–F63. <https://doi.org/10.1111/1468-0297.00098>
- Kumbhakar, S. C., & Tsionas, E. G. (2011). Some recent developments in efficiency measurement in stochastic frontier models. *Journal of Probability and Statistics*, 2011, Article 603512. <https://doi.org/10.1155/2011/603512>
- Kumbhakar, S. C., Wang, H.-J., & Horncastle, A. P. (2015). *A practitioner's guide to stochastic frontier analysis using Stata*. Cambridge University Press.
- Lavy, V. (2015). Do differences in schools' instruction time explain international achievement gaps? *Economic Journal*, 125(588), F397–F424. <https://doi.org/10.1111/econj.12233>
- Lavy, V., Paserman, M. D., & Schlosser, A. (2012). Inside the black box of ability peer effects: Evidence from variation in the proportion of low achievers in the classroom. *Economic Journal*, 122(559), 208–237. <https://doi.org/10.1111/j.1468-0297.2011.02463.x>
- Levačić, R., & Vignoles, A. (2002). Researching the links between school resources and student outcomes in the UK: A review of issues and evidence. *Education Economics*, 10(1), 1–22.



Original Article

- <https://doi.org/10.1080/09645290110115776>
- Lou, Z. (2024). Integrated production and efficiency analysis in higher education: A multi-output frontier approach. *Higher Education*, 87(3), 511–533. <https://doi.org/10.1007/s10734-023-01041-2>
- Manski, C. F. (1993). Identification of endogenous social effects: The reflection problem. *Review of Economic Studies*, 60(3), 531–542. <https://doi.org/10.2307/2298123>
- Meeusen, W., & van den Broeck, J. (1977). Efficiency estimation from Cobb–Douglas production functions. *International Economic Review*, 18(2), 435–444.
- Mincer, J. (1958). Investment in human capital and personal income distribution. *Journal of Political Economy*, 66(4), 281–302. <https://doi.org/10.1086/258055>
- Mullainathan, S., & Spiess, J. (2017). Machine learning: An applied econometric approach. *Journal of Economic Perspectives*, 31(2), 87–106. <https://doi.org/10.1257/jep.31.2.87>
- OECD. (2001). *Measuring productivity: OECD manual*. OECD Publishing.
- OECD. (2018). *The future of education and skills: Education 2030*. OECD Publishing.
- Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., et al. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *BMJ*, 372, n71. <https://doi.org/10.1136/bmj.n71>
- Ranasinghe, A. (1999). Measurement of efficiency in Sri Lankan education: A joint production perspective. *Sri Lanka Journal of Social Sciences*, 22(1–2), 45–62.
- Rao, V. V. B. (1969). Two examples of joint production. *Review of Economic Studies*, 36(1), 93–96. <https://doi.org/10.2307/2296410>
- Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.), Sage Publications.
- Rivkin, S. G., Hanushek, E. A., & Kain, J. F. (2005). Teachers, schools, and academic achievement. *Econometrica*, 73(2), 417–458.
- Rolle, A. (2004). *The economics of education: Production and efficiency in schools*. Information Age Publishing.
- Romer, D. (1993). Do students go to class? Should they? *Journal of Economic Perspectives*, 7(3), 167–174. <https://doi.org/10.1257/jep.7.3.167>
- Rothstein, J. (2010). Teacher quality in educational production: Tracking, decay, and student achievement. *Quarterly Journal of Economics*, 125(1), 175–214. <https://doi.org/10.1162/qjec.2010.125.1.175>
- Sacerdote, B. (2001). Peer effects with random assignment: Results for Dartmouth roommates. *Quarterly Journal of Economics*, 116(2), 681–704. <https://doi.org/10.1162/00335530151144131>
- Simar, L., & Wilson, P. W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics*, 136(1), 31–64. <https://doi.org/10.1016/j.jeconom.2005.07.009>
- Stanca, L. (2006). The effects of attendance on academic performance. *Journal of Economic Education*, 37(3), 251–266. <https://doi.org/10.3200/JECE.37.3.251-266>
- Sullivan, D. F., Mackie, C., Massy, W. F., & Sinha, E. (2012). *Improving measurement of productivity in higher education*. National Academies Press. <https://doi.org/10.17226/13417>
- Summers, A. A., & Wolfe, B. L. (1977). Do schools make a difference? *American Economic Review*, 67(4), 639–652.
- Sun, Y. (2023). Efficiency evaluation of higher education institutions using a double-frontier DEA model. *Applied Economics*, 55(18), 2065–2083. <https://doi.org/10.1080/00036846.2022.2108674>



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- Todd, P. E., & Wolpin, K. I. (2003). On the specification and estimation of the production function for cognitive achievement. *Economic Journal*, 113(485), F3–F33. <https://doi.org/10.1111/1468-0297.00097>
- Vinod, H. D. (1968). Canonical ridge and econometrics of joint production. *Journal of the American Statistical Association*, 63(322), 506–515. <https://doi.org/10.1080/01621459.1968.10480820>
- Wedel, M. (2021). Model specification and functional form uncertainty in applied econometrics. *Journal of Economic Methodology*, 28(4), 375–392.
- Zimmer, R., & Toma, E. F. (2000). Peer effects in private and public schools. *Journal of Policy Analysis and Management*, 19(1), 75–92. [https://doi.org/10.1002/\(SICI\)1520-6688\(200024\)19:1](https://doi.org/10.1002/(SICI)1520-6688(200024)19:1)